Time-domain versus frequency-domain effort weighting in active noise control

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Abstract

Although Active Noise Control aims at reducing the noise at a set of error sensors, it is often designed by minimizing an error index that includes a weighted penalty on the actuator inputs. In this way, the control tends to be more robust and the effort-weighting parameter allows the maximum voltages applied to the control sources to be monitored. Two similar effort-weighting techniques have been widely implemented in active control studies: optimal control can be computed using Tikhonov regularization in frequency-domain simulations, whereas the leaky Filtered-reference least mean squares algorithm can be implemented for real-time feedforward control. This paper makes explicit the relationship between the two effort-weighting parameters which lead, in the case of a single-tone noise, to exactly the same error index in both the time and frequency domains. The best real-time leakage factor can then be computed from frequency-domain optimization. This paper also discusses numerical simulations of a single-channel set-up, showing that with these two related parameters, the control performances are indeed very similar. One exception occurs in the case of a control filter with

a very short impulse response, when the control is more conservative in the time-domain simulations than in those of the frequency-domain.

1 Introduction

Loudspeakers or shakers can be destroyed when they are driven by a high voltage. To prevent this risk, in situations where the primary noise to be reduced requires high voltages at the secondary actuators, an Active Noise Control set-up is usually designed so as to minimize an error index including both the control signals (the actuator inputs) and the error signals (the residual noise at the error sensors) (*cf.* [Nelson(1992)], [Elliott(2001)], [Kuo(1996)]). By adjusting a weighting coefficient in the error index, the trade-off between noise reduction and voltage limitation can be optimized.

From a theoretical point of view, the inclusion of an effort-weighting parameter in the minimization index also allows ill-posed control problems (e.g. in situations where there are more actuators than error sensors) [Tikhonov(1977)] to be regularized. It has also been reported [Nelson(1992)] that effort weighting in active control may enlarge the area in which noise is reduced, because it forces noise minimization at a discrete set of sensors to be better matched to an underlying, continuous-space, global control problem. Finally, leakage has the virtue of increasing the robustness of a real-time adaptive algorithm to errors in the secondary path estimation. This is a useful property for on-line computations (see [Elliott(2001)], section 3.4.7). Therefore, besides monitoring the control inputs in the case of a loud primary noise, it is generally a good idea to include an effort-weighting parameter in the design of an active control set-up.

Most active control studies address the weighting of the control inputs through the use of two different techniques:

1. at the design stage, the control signals and the noise reduction are usually computed off-line, in frequency domain simulations. In this case the actuator inputs can be limited by minimizing, at each frequency, a quadratic index which weights the norm of the sensor signal vector and the norm of the actuator input vector (see [Elliott(2001)], section 4.2.6). Optimal control can then be determined by performing the regularized inversion of the theoretical or measured actuator-to-sensor response matrix [Tikhonov(1977)].

2. For effective real-time control, the actuator inputs are usually computed on-line in the time domain, using an adaptive algorithm such as the Filtered Reference Least Mean Square algorithm (FxLMS). The FxLMS constantly adapts the coefficients of the transverse Finite Impulse Response (FIR) control filters which generate the control signals. In this case, it is straightforward to take the effort weighting into account, by including the norm of the FIR filter coefficients in the error index. This leads to the introduction of a so-called leakage factor (*cf.* [Elliott(2001)], section 3.4.7) in the FxLMS updating formula.

These approaches are similar. They both include a quadratic vector norm of residual noise in the error index and, in the time domain, the leakage factor monitors the trade-off between control inputs and performance, as does the regularization parameter in the frequency domain. In the case of a single-tone noise, these approaches are exactly equivalent, because a change in magnitude in the control filter coefficients leads to the same change in the control signals. However, no obvious relationship has been established between the two approaches, even though it would be advantageous if the leakage factor to be implemented in real-time could be deduced from preliminary frequency-domain simulations. Indeed, although a trial-and-error procedure can be used in the latter simulations, in order to determine the best effort-weighting parameter, the leakage factor cannot be optimized in the time domain without running the risk of damaging the actuators.

The present paper provides, in the case of a single-tone noise, the theoretical relationship between the regularization parameter in the frequency domain, and the leakage factor in the time domain, which both lead to the minimization of the same error index. Using this relationship, the leakage factor to be used on-line can be computed from a regularization parameter selected by means of frequency-domain simulations.

This paper also presents numerical simulations of FxLMS control for a simple acoustic set-up, in which the leakage coefficient has been computed from the frequency-domain regularization parameter. The simulations show that the real-time control leads to slightly lower control signals and performance than predicted in the frequency domain. The gap between the respective control results becomes narrower, as the length of the control FIR is increased.

In section 2 of this paper, the two conventionally used techniques for effort weighting in active noise control are briefly recalled. Notations are introduced, enabling the subsequent derivation of the correspondence between these two techniques. The relationship between the regularization parameter and the leakage factor is derived in section 3. Section 4 presents numerical simulations of a simple control set-up, showing that when the corresponding minimization indexes are used, control with a leaky FxLMS algorithm is slightly more conservative in terms of control input limitations than the direct frequency domain optimization.

2 Weighted error indexes and optimal control

Fig. 1 is a drawing of a typical feedforward active noise control set-up. The reference signal, x, which is correlated with the noise to be reduced by the active control, drives a set of linear filters with vector $\mathbf{w}(\omega)$ of Frequency Response Functions (FRF) at the angular frequency ω. $\mathbf{u}(\omega)$ is the vector corresponding to the control filter outputs, which are the actuator inputs, and $\mathbf{e}$ is the vector describing the noise at the set of error sensors. $\mathbf{e}_p$ is the so-called primary noise at the sensors, i.e. the noise without control. Matrix $\mathbf{H}(\omega)$ denotes the so-called secondary path matrix of the FRFs between the actuator inputs and the error sensors, at the angular frequency ω. Similarly, the vector $\mathbf{f}(\omega)$ denotes the primary path between the reference signal x and the primary noise $\mathbf{e}_p$.

2.1 Optimal control in the frequency domain

Active control at the angular frequency ω can easily take effort weighting into account by using the minimization index (see [Elliott(2001)], section 4.2):

$$J_\omega = \|\mathbf{e}(\omega)\|^2 + \gamma^2 \|\mathbf{u}(\omega)|^2 \tag{1}$$

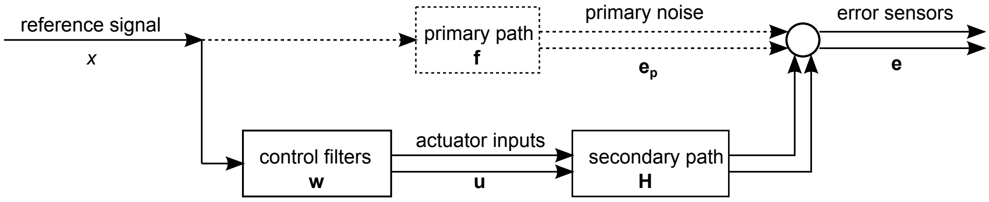

Figure 1: A feedforward active noise control set-up

where $\| \bullet \|$ is the usual vector 2-norm and γ^2 is the effort-weighting parameter. Index 1 can be expanded as:

$$J_\omega = \|\mathbf{H}(\omega)\mathbf{u}(\omega) + \mathbf{e}_p(\omega)\|^2 + \gamma^2\|\mathbf{u}(\omega)\|^2 \tag{2}$$

The input signal vector which minimizes this index is then given by (*cf.* [Tikhonov(1977)]):

$$\hat{u}(\omega) = \mathbf{H}^\dagger \mathbf{e}_p(\omega) \tag{3}$$

where $\mathbf{H}^\dagger$ is the so-called Tikhonov-regularized pseudo-inverse matrix of $\mathbf{H}(\omega)$. $\mathbf{H}^\dagger$ can be computed by inverting $\mathbf{H}(\omega)$ through Singular Value Decomposition, and by replacing every singular value $\frac{1}{\sigma_i}$ of the inverse matrix $\mathbf{H}$ by $\frac{\sigma_i}{\sigma_i^2+\gamma^2}$. General purpose scientific computing packages include routines for regularized matrix inversion, such as function *pinv* in Matlab^{R}®.

2.2 Optimal control in the time-domain

According to [Elliott(2001)] (section 3.4.7), effort weighting can be taken into account in real-time FxLMS control by minimizing the error index:

$$J_n = \|\mathbf{e}(n)\|_n^2 + \nu\|\mathbf{w}(n)\|_n^2 \tag{4}$$

where

- $\mathbf{e}(n)$ is the vector of the instantaneous error signals at discrete time n and $\|\mathbf{e}(n)\|_t$ is its mean square value over time,
- $\mathbf{w}(n)$ is the stacked vector of all the control FIR at time n,
- ν is an arbitrary effort-weighting coefficient.

The filter responses which minimize index J_t can be computed using the adaptive form:

$$\mathbf{w}(n+1) = (1-\nu\beta)\mathbf{w}(n) - \beta\mathbf{R}(n)e(n) \tag{5}$$

where

- β is a proper convergence coefficient
- $\mathbf{R}(n)$ is the so-called *filtered reference*, providing filtering of the reference signal $x(n)$ through FIR estimates of the secondary path.

The updating formula 5 is constitutive of the so-called leaky FxLMS algorithm.

3 Linking the effort-weighting parameters

In this section, the error indexes 1 and 4 are manipulated in order to determine the values of the weighting parameters γ and ν which lead to the same minimization problem.

At first, as a consequence of Parseval's theorem, the error signal contributions have the same form in both indexes, as soon as the FxLMS has converged to the steady-state optimal control. The time-domain minimization index can thus be rewritten:

$$J_n = \|\mathbf{e}(\omega)\|^2 + \nu\|\mathbf{w}(n)\|_n^2 = \|\mathbf{e}(\omega)\|^2 + \nu\frac{\|\mathbf{w}(n)\|_n^2}{\|\mathbf{u}(\omega)\|^2}\|\mathbf{u}(\omega)\|^2 \tag{6}$$

By comparing Eq. (4) and Eq. (6), it can be seen that both minimization indexes will lead to the same solution if, and only if:

$$\nu = \frac{\gamma^2\|\hat{\mathbf{u}}(\omega)\|^2}{\|\hat{\mathbf{w}}\|^2} \tag{7}$$

where $\hat{\mathbf{w}}$ is the stacked vector of the *optimal* control filter FIRs.

Eq. (7) formally connects the two effort-weighting factors but vector $\|\hat{\mathbf{w}}\|$ in 7 is not a direct result computation of optimal control in the frequency domain. However, in the frequency domain, the FRFs $\hat{\mathbf{w}}(\omega)$ of the optimal filters are easy to compute, because the optimal control inputs $\hat{\mathbf{u}}$ are the filtering of the reference signal:

$$\|\hat{\mathbf{w}}(omega)\| = \frac{\|\hat{\mathbf{u}}(\omega)\|}{\sqrt(2)\|x(n)\|_n} \tag{8}$$

The factor $\sqrt(2)$ used in this formula arises from the definition of $\|x(n)\|_n$, as the quadratic mean square value of the reference signal $x(n)$.

The FIRs of the optimal control filters can now be determined from the FRFs by using the fact that, among all the FIR filters with frequency responses $\hat{\mathbf{w}}(\omega)$, the optimal control FIR vector has a minimum vector 2-norm because it minimizes index 4.

In the case of a single actuator, the optimal FRF $\hat{\mathbf{w}}(\omega)$ is a single complex number and $\hat{\mathbf{w}}$ is a single FIR $[\hat{w}_0 \; \hat{w}_1 \; \ldots \& \hat{w}_{N-1}]^t$, where N is the FIR length, of the form:

$$\hat{\mathbf{w}}(\omega) = \hat{\mathbf{w}}_0 + \hat{w}_1 e^{-i\omega/f_s} + \ldots + \hat{w}_{N-1}e^{-i(N-1)\omega/f_s} = \left[1 \; e^{-i\omega/f_e} \ldots e^{-i(N-1)\omega/f_s}\right]^t \hat{\mathbf{w}} \tag{9}$$

In the above expression, f_s is the sampling frequency of the real-time control. The vector $\hat{\mathbf{w}}$ is theferore the least squares solution of the linear equation:

$$\begin{bmatrix} \Re(\hat{\mathbf{w}}(\omega)) \\ \Im(\hat{\mathbf{w}}(\omega)) \end{bmatrix} = \begin{bmatrix} 1 & \dots & \cos((N-1)\omega/f_s) \\ 0 & ldots & \sin((N-1)\omega/f_s) \end{bmatrix}^t \hat{\mathbf{w}} \tag{10}$$

This can, for example, be solved in Matlab® code by a matrix division using the symbol \.

It should be noted that the FIRs, $\hat{\mathbf{w}}$, and therefore the computed leakage factor, ν, depend strongly, but not in a straightforward manner, on the length N of the control filter FIRs. Real-time changes in the control filter thus lead to the need for changes in the leakage factor, if the effort weighting is to be maintained at the same level.

In the multiple-actuator case, the stacked vector $\hat{\mathbf{w}}$ of optimal FIRs is obtained in the same way, by stacking the optimal control FRFs on the left-hand side of Eq. (10).

To summarize, the leakage factor corresponding to a given regularization parameter γ can be computed using the following procedure:

- compute the optimal control inputs $\hat{u}(\omega)$ by means of a pseudo-inversion, as discussed in section 2.1
- compute the FRFs of the optimal filters $\hat{\mathbf{w}}(\omega)$ using Eq. (9)
- compute the FIRs of the optimal filters $\hat{\mathbf{w}}$ using a least-square inversion (Eq. (10))
- compute the leakage factor from Eq. (7):

$$\nu = \frac{\gamma^2 \|\hat{\mathbf{u}}(\omega)\|^2}{\|\hat{\mathbf{w}}\|^2} \tag{11}$$

4 Time-domain vs. frequency-domain simulations

The above procedure was implemented for the numerical simulation of a very simple, single-channel control case.

The primary noise was a 300Hz pure-tone $e_p(t) = \pi \sin(2\pi 300t)$. The reference signal was taken as $x = 3\sqrt{2}\sin(2\pi 300t)$, which was perfectly correlated with the primary noise. FxLMS was simulated at the sampling frequency $f_s = 1000Hz$. The secondary path was chosen as a pure two-sample delay with FIR $\mathbf{h} = [0\ 0\ 1]$.

For the results given below, the FxLMS convergence coefficient (β in Eq. (5)), normalized by the control filter FRF length and the mean square value of the reference (*cf.* [Elliott(2001)], section 3.4.4), was assigned to 0.4. The mean-square values of the error and control signals were computed over a 2 second period, after running the FxLMS for 8 seconds, which was much longer than the apparent control convergence time.

Table 1 shows the resulting control performances for a set of regularization parameters γ and control FIR lengths N. u_ω and u_n are the mean-square values of the control signal in the frequency and time domains, respectively, and $\mathcal{A}_\omega$ and $\mathcal{A}_n$ are the corresponding values of noise attenuation.

Firstly, table 1 shows that, as expected, the regularization monitors the control signal; the greater the effort weighting, the smaller the control signal and noise attenuation.

Secondly, table 1 shows that the regularization parameter γ and the leakage factor ν, computed as discussed in section 3, lead to a control signal magnitude and a noise attenuation that are very close. This result confirms, if necessary, that the procedure discussed in section 3 gives effort-weighting parameters leading to the same minimiza-

γ	N	u_{freq}	u_{temp}	$\mathcal{A}_\omega$ (dB)	$\mathcal{A}_n$ (dB)
0	2	0.7405	0.7407	$+\infty$	223
0.05	2	0.7386	0.7099	52.1	27.2
0.05	10	0.7386	0.7385	52.1	50.7
0.05	50	0.7386	0.7385	52.1	52.1
0.2	2	0.7120	0.4357	28.3	7.20
0.2	10	0.712	0.7076	28.3	27.0
0.5	2	0.5924	0.1818	14.0	1.1
0.5	10	0.5924	0.5666	14.0	12.6
1	10	0.3702	0.2920	6.0	4.3

Table 1: Optimal control indexes related to regularization parameter γ and control FIR length N

tion problem. However, the control in the time-domain is slightly more conservative when the control filter FIR is very short in length. This is probably due to the lack of accuracy of the FxLMS, when the selected length of the control filter FIR is too short.

5 Concluding remarks

In the case of a single-tone primary noise, Tykhonov regularization in the frequency domain and leaky-FxLMS active control are exactly equivalent. A simple numerical procedure has been introduced in this paper to compute the leakage factor corresponding to a given regularization parameter; simple numerical simulations have confirmed that the two effort-weighting approaches lead to almost the same control results.

In the case of a primary noise involving several tones, the convergence of the FxLMS algorithm can be considered independently at each frequency. A single leakage factor $\nu(\omega_k)$ could thus be computed from the regularization parameters $\gamma(\omega_k)$, for each frequency. However, since Eq. (10) is frequency dependent, a single global leakage factor cannot be computed, even if the frequency-domain parameter is the same at all frequencies. In the case of broadband noise, control cannot be computed independently at each frequency, as a consequence of the causality constraint (*cf.* [Elliott(2001)]); in this case, as discussed in section 2.1, even optimal frequency-domain control cannot be easily computed in the multi-channel case.

Finally, although this paper has focused on active noise control, real-time effort-weighting is of interest for the solution of other problems in acoustics. For example, so-called Adaptive Wave-Field Synthesis, which was introduced for the purposes of 3D sound field generation, involves the on-line minimization of an index including a quadratic penalty [Gauthier(2008)]. In this case, the time-domain regularization parameter can also be computed from the frequency-domain parameter, as discussed in section 2, for the case of single-tone synthesis.

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